



A Hamiltonian numerical scheme for a hydrostatic atmosphere

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Although our climate is ultimately driven by (nonuniform) solar heating, many aspects of the flow can be understood qualitatively from forcing-free and frictionless dynamics. In the limit of zero forcing and dissipation, our weather system falls under the realm of *Hamiltonian fluid dynamics* and the flow conserves potential vorticity, energy and phase space structure.

We have found a conservative numerical scheme for an adiabatic and hydrostatic atmosphere. The steps are as follows. First, we derive a mixed Eulerian-Lagrangian Hamiltonian description of the continuum atmosphere based on a so-called *parcel formulation*, [1.]. Key point here is that we use isentropic coordinates. The next step is to introduce a finite element discretization in the entropic direction. What remains is a stack of shallow water-type flows that are coupled through an elliptic equation in the entropy coordinate only. Next, we use a *Hamiltonian Particle-Mesh* method, [2], to discretise the horizontal dynamics. This method is based on a discrete version of the parcel formulation and uses a symplectic time integrator. Potential vorticity as well as Hamiltonian phase space structure are exactly conserved. Energy is conserved asymptotically in the time step.

Several challenging nonlinear solutions will be tested.

A successful application of our method can be found in [3.], where a simplifying *layer-model* approximation is used in the entropic direction.

- 1 Bokhove, O. & Oliver, M. (2006): 'Parcel Eulerian-Lagrangian fluid dynamics for rotating geophysical flows'. *Proc. Roy. Soc. A.* **462**, 2563–2573.
2. Frank, J., Gottwald, G., Reich, S. (2002): 'A Hamiltonian particle-mesh method

for the rotating shallow-water equations'. *Lecture Notes in Computational Science and Engineering*, vol. **26**. Springer, Heidelberg, 131–142.

3. Bokhove, O. & Oliver, M. (2008): 'Isentropic N-layer models for atmospheric dynamics'. Accepted subject to minor modifications *Q. J. Roy. Met. Soc.* & <http://eprints.eemcs.utwente.nl/>