



Data Assimilation in the guaranteed uncertainty estimation. Affine Arithmetics and Kalman-like filter

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The guaranteed uncertainty problems often arise in the sensitivity analysis and parameter estimation of hydrological models. Additionally the guaranteed error estimation is more preferable to more popular L2 error estimation in decision support systems applications.

The propagation of model guaranteed uncertainty can be done quite straightforward with the help of Interval Arithmetics. Unfortunately Interval Arithmetics suffers from precision loss in long calculation and, what is more important, does not preserve correlations between quantities by treating all uncertainties being mutually independent. Noting that in Data Assimilation algorithms a correlation level between two corresponding variables plays a crucial role in forming a state vector update, one has to search for Interval Arithmetics branches that explicitly suggest computation of "common" uncertainty of any two variables.

Affine Arithmetics [1] is an extension of Interval Arithmetics, which like standard interval arithmetic (IA) can provide guaranteed bounds for the computed results, taking into account input, truncation, and rounding errors. Unlike interval arithmetic, it keeps track of correlations between computed and input quantities, and is therefore resistant to the catastrophic loss of precision often observed in long interval computations.

A Kalman-like filter is presented utilizing correlations, automatically computed by rules of the studied computational model and the corresponding Affine Arithmetics counterparts of standard mathematical operations. Filter degrades to one update equation leaving the uncertainty tracking to the computational model in its Affine Arith-

metics realization. C++ version of the filter is realized with libaffa library [2].

An important observation for the real life applications lies in the fact that having a computational model realized in C++ language in order to make it do computations in Affine Arithmetics it is sufficient to change declaration of all `*double*` variables to `*AAF*` variables with some additional minor changes.

Performance of the filter is shown on its application to Advection-Diffusion equation.

[1] Affine arithmetic and its applications to computer graphics. Joao L. D. Comba and J. Stolfi. Presented at SIBGRAPI'93, Recife, PE (Brazil), October 20-22, 1993.

[2] <http://www.nongnu.org/libaffa/>