Statistical Modeling of the Spatial Dependence between annual maxima of precipitation

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Climatological or pollutant data are recorded at spatial locations, $x_i, i = 1, \ldots, N$, and the measurements at different locations $X_i, i = 1, \ldots, N$ exhibit dependence. Given data such as daily precipitation or daily pollutant levels, it is well understood how to estimate the data's spatial dependence and how to perform spatial prediction. The spatial dependence structure is usually estimated using the variogram, $\frac{1}{2}E\left[(X_i - X_j)^2\right]$ because of its relationship with covariance. Spatial prediction is generally done via Kriging and the predictor for location x_0 is given by $\hat{X}_0 = \sum_{i=1}^{N} \lambda_i X_i$. The weights λ_i are determined by minimizing the loss function $E\left[(\hat{X}_0 - X_0)^2\right]$ with the condition that $\sum_{i=1}^{N} \lambda_i = 1$. It is often assumed that data such as daily precipitation come from an underlying Gaussian process. When a Gaussian process is assumed, then the dependence structure is completely determined by the variogram, and the Kriging predictor $\hat{X}_0$ also has a Gaussian distribution.

However, when the data to be modeled are maxima, such as annual maximum daily precipitation or annual maximum pollutant level, it is not understood how to model the spatial dependence or to perform spatial prediction. The dependence structure of annual maxima is more difficult than daily data; for instance the annual maximum at one location may occur on a different day than the annual maximum at a nearby location. We cannot assume an underlying Gaussian distribution for annual maximum data, and instead assume that the distribution comes from the family of extreme value distributions. The family of multivariate extreme value distributions cannot be described parametrically, which further complicates matters.

We propose estimating the spatial dependence between extreme values with the mado-

gram, $\frac{1}{2}E\,|X_i - X_j|$, and present its convenient relationship with the extremal coefficient, a measure of dependence between extreme value random variables. We present a spatial predictor for the annual maximum at location x_0 of $\hat{X}_0 = \bigvee_{i=1}^{N} \lambda_i X_i$. By taking a maximum of weighted observations the distribution of the predictor remains in the family of extreme value distributions.

In this poster we will present our reasoning for using the madogram and the max-kriging predictor. In addition, we will present some preliminary results which illustrate the madogram's usefulness in estimating spatial dependence and the max-kriging predictor's usefulness for spatial prediction.