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Mutivariate bilinear time series; a stochastic alternative in population dynamics

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The general form of a multivariate bilinear time series model is given by

$$X(t) = \sum A_i \cdot X(t-i) + \sum M_j \cdot e(t-j) + \sum\sum\sum B_{dij} \cdot X(t-i) \cdot e_d(t-j) + e(t).$$

Here the state $X(t)$ and noise $e(t)$ are n-vectors and the coefficients A_i, M_j, and B_{dij} are n by n matrices. If all $B_{dij} = 0$, we have the class of well-known vector ARMA-models. The bilinear models include additional product terms $B_{dij} \cdot X(t-i) \cdot e_d(t-j)$; as the name indicates these models are linear in state $X(t)$ and in noise $e(t)$ separately, but not jointly. From a theoretical point of view, it is therefore natural to consider bilinear models in the process of extending existing linear theory to non-linear cases. But there are at least equally good practical reasons. Continuous-time deterministic bilinear models were studied in control theory in the 1960s. A large number of applications, also on ecology, made it natural to consider stochastic analogues of these deterministic models.

Extensions have been made, first to univariate and then to multivariate bilinear models. The main results give conditions for stationarity, ergodicity, invertibility, and consistency of least square estimates. They are stated as inequalities that involve matrix norms and expectations of stochastic terms. Computer programs have been written to find the estimates of the coefficient matrices from observed values of the state variable $X(t)$. These have been used for simulation studies of the distribution and robustness of the estimates.

A particular reason for introducing bilinear time series in population dynamics, is that they are suitable for modelling environmental noise. One may start with a deterministic system with (constant) parameters that describe conditions that depend on a fluctuating environment. The idea is to replace them by stochastic parameters. As an example we consider the familiar prey-predator system

$$\frac{dN_1}{dt} = r_1 \cdot N_1 \cdot (1 - a \cdot N_1 - b \cdot N_2), \quad \frac{dN_2}{dt} = r_2 \cdot N_2 \cdot (-1 + c \cdot N_1 - d \cdot N_2)$$

with positive parameters, $c > a$. The Lotka-Volterra specialization $a = d$=0 is known to be structurally unstable and all nontrivial solutions are cyclic. If $(a + d)$ >0 and (ad+bc)>0 there is a local asymptotically stable equilibrium (N_1^*, N_2^*).

Linearizing about this equilibrium, replacing derivatives by differences, and adding noise terms in the coefficients lead to a single lag bilinear bivariate (i.e. n=2) model.

$$\begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} X_1(t-1) \\ X_2(t-1) \end{bmatrix}$$
$$+ \begin{bmatrix} b_{111} & b_{112} \\ b_{121} & b_{122} \end{bmatrix} \cdot e_1(t-1) \cdot \begin{bmatrix} X_1(t-1) \\ X_2(t-1) \end{bmatrix}$$
$$+ \begin{bmatrix} b_{211} & b_{212} \\ b_{221} & b_{222} \end{bmatrix} \cdot e_2(t-1) \cdot \begin{bmatrix} X_1(t-1) \\ X_2(t-1) \end{bmatrix}.$$